DIMENSIONALITY REDUCTION IN OPTIMAL OPERATION OF HYDRO-THERMAL POWER-GENERATION DYNAMIC SYSTEMS

RESEARCH ADVANCES
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Plan of the presentation

- Brief introduction to optimal operation of dynamic systems.
- The Bellman Curse of Dimensionality.
- The power generation as a Dyn.Sys.
- Kind of state variables.
- The SimSEE platform as a Lab. For Red-Amp transformations test.
Brief introduction to optimal operation of dynamic systems.

- Dynamic System (State and Inputs)
- Evolution equation.
- Optimal Policy
- Optimization – Simulation.
- SDP
- The curse dimensionality of Bellman
- A bit on DSDP
Discretization of the time horizon
Discrete Time Dynamic Systems

$x_k \in \mathbb{R}^n$ Is the STATE of the system an capture all the information from the past to compute the future evolution knowing the inputs.

$r_k \in \mathbb{R}^{n_r}$ Are the non-controllable inputs.

$u_k \in \mathbb{R}^{n_u}$ Are the controllable inputs.

$k$ Identify the time stage.
The STATE is the information from the past that is necessary to know for computing the future evolution of the system, knowing the future inputs.

If you know the state value, you know all from the system's past that has influence in the system's future.

$$x_{k+1} = f(x_k, r_k, u_k, k)$$
Driving the system

The system is driven by the control variables $u$ and by the uncontrolled random inputs $r$
Operation Policy

- An Operation Policy is a map between the state and the non-controllable inputs that for every stage $k$ gives the values of the controllable inputs to be applied to the system.

$$u_k = po(x_k, r_k, k)$$
$u_k = po(x_k, r_k, k)$
The cost of the future (CF) and the cost to go (CE)

\[ CF(x, U_k, R_k, k) = \sum_{j=k}^{\infty} q^{j-k} \cdot CE(x_j, u_j, r_j, j) \]

\[ U_k = \{u_k, u_{k+1}, \ldots\} \]

\[ R_k = \{r_k, r_{k+1}, \ldots\} \]
Optimal operation

\[ CF(x, k) = \min_{u_k} \{ CE(x, u_k, r_k, k) + q \cdot CF(x', k+1) \} \]

\[ x' = f(x, u_k, r_k, k) \]
\[ U_k = \{ u_k, u_{k+1}, \ldots \} = \{ u_k, U_{k+1} \} \]
\[ R_k = \{ r_k, r_{k+1}, \ldots \} = \{ r_k, R_{k+1} \} \]
Optimization of each stage

\[ t_{1} + k x_{k} u_{k}(t) + \sum_{i=1}^{r} k x_{k} CF_{CE}(i) + \sum_{j=1}^{r'} k x_{k} CF_{CE'}(j) \]
The Policy is in the derivatives of $dCF(x,k) / dx$

- For each stage $k$ the operator needs the representation of $CF$ in the domain of the state $x$. 
SDP Recursion

para todo \( x \) hacer:
\[
CF(x, k_{ultima+1}) = 0
\]

para \( k \) desde \( k_{ultima} \) retrocediendo hasta 1 hacer:

para todo \( x \) hacer:
\[
CF(x, k) = \min_{u_k} \left\{ CE(x, u_k, r_k, k) + q \cdot CF(x', k+1) \right\}_{r_k}
\]

con \( x' = f(x, u_k, r_k, k) \)
y sujeto a \( g(x, u, r, k) \leq 0 \)
The Bellman Curse of Dimensionality

$$x_k^1 + k x_2^1 + k x_1^k (kT) + kT$$

$$N_1 \times N_2 \times ... \times N_{nx}$$
Two well known strategies to face this optimization problem

• Stochastic Dynamic Programming (SDP)
• Stochastic Dual Dynamic Programming (SDDP).
Stochastic Dynamic Programming (SDP)

- The SDP computes the cost function from the future back to the present.
- To proceed with the calculus, a discretization, both in time and space, is defined for each of the state variables of the system.
- This leads to the well known Bellman’s "curse of dimensionality" that turns the SDP not applicable when the number of state variables increases.
SDDP vs. the Curse of dimensionality.

- The SDDP leads with the dimension of the state space using Benders cuts to approximate the cost function for each time step by hyper-planes in the state-space.

- The approximation is carried out in successive sweeps of the stages forward, computing the cost of a feasible solution and backward computing the cost of the relaxation.

- If stochastic inputs are present, the process opens on a tree of approximations that may suffer of a sort of “curse of dimensionality”.

- Very good method for large system with large number of reservoirs, using main values for stochastic inputs.
SDDP vs. convexity

• If the cost function and the constrains are convex, we obtain the exact solution. Without convexity, we have a gap, “the duality gap”.

• When the production costs of the fuel fired units are considered constant, the resulting cost functions are convex, linear, so the overall production cost is also convex and the method is applicable. When a more detailed production cost function is considered, a minimum operation point appears resulting in a non convex function.

• If the system is great enough the duality gap is irrelevant. But in small systems, where the power of a unit is greater than 10% of the power of the demand the duality gap may be relevant.

• Very good method for large system with large number of reservoirs, using main values for stochastic inputs.
we choose classical SDP

- The daily maximum of the power demand in Uruguay is about 1300MW.

- The greatest thermal unit in the system has a power of 125MW, so the system is very small and some care must be taken with dual optimization techniques.

- The 60% of the energy comes from one hydro plant and so the stochastic modeling of the water inflows is important.

- It is also true that classical SDP method are more suitable for distributed programming and with the permanent increasing of the power of computers at lower prices, it is foreseeable that SDP method can be implemented in spite of ”the curse of dimensionality”.
operation of an hydrothermal power system

a complex optimization problem
Power Systems.

- Generation Plants
- Hydroelectrics – the reservoir
- Fuel fired – Steam Boiler cost ON/OFF and time to startup.
Using the stocked water

The complexity comes from the fact that we are leading with systems with reservoirs.

The problem is not only how much to use of each of the stocks but also when to use them.
Cost of operation

FUEL consumption at the fuel fired power plants. Imports from other countries FAILURE in supplying energy to the system load.
• The use of stocked water today potentially increases the cost of stages in the future. The preservation of water today for a later use may reduce the cost of some stages in the future, but really increases the cost today due to the additional fuel based generation.

The problem is to find a policy of use of the stocked resources that results in an equilibrium between present and future costs.
Hey, do not sleep!
Uruguay Power System
Long – term representation

- Volume in lake of “Rincón de Bonete”'s dam.
- Hydrological condition.

![Graph](image)

CF frame for a given k
CF( V, H, k)
Remember the information is in the derivatives!
Red-Amp Transform

- \( Z = \text{Red}( X ) \)
- \( X = \text{Amp}( Z ) + \text{Noise} \)
- \( \text{Red}( \text{Amp}( Z ) + \text{Noise} ) = Z \)
- Reconstruct state probabilities X
Kind of state variables

- We can act over the Stored Resources using them.
- We can not act over the hydrological condition.
Kind of state variables

- We can act over the Stored Resources using them.
- We can not act over the hydrological condition.
Reduction by One Frame Computation

- One of the research lines is in the automatic reduction from a frame of CF computed without reduction of state space. It is achieved looking for the directions of great derivatives.

- Then the SDP continues using the reduced state space but some test points are computed in the “blind dimension” to determine when another whole frame needs to be computed.
Brute Force Reduction

- Given a Reduction Transformation and an Amplification Transformation we can simulate a lot of realizations of the system's behavior and measure the “real” cost of operation. The less the cost, the better the pair of Red_Amp Transforms.
- We are using a distributed simulator based on Genetics Algorithms with “intelligent agents” to perform these kind of Red_Amp Computations.
- This line of research is promising because the system (the power system) is always the same and is running all the time to program its operation and for planning the installation of future plants.